Shortfall on a strip of options

Tom Weston

May 27, 2020

We quote a formula for the shortfall on a strip of options,

$$\sum_{i=1}^{M} (F_i - X)^+ \tag{1}$$

where

- X is a strike
- σ is the volatility
- F_0 is the forward
- τ is the time to expiry
- α is the chosen percentile
- $Z(x) = e^{-x^2/2}/\sqrt{2\pi}$
- $N(x) = \int_{-\infty}^{x} Z(x')\mathrm{d}x'$
- M is the number of (identical) options

$$\begin{aligned} ES(\alpha) \quad = \quad & \sqrt{M}\left(\frac{Z[N^{-1}(\alpha)]}{1-\alpha}\right)\left\{F_0^2\left[\exp(3\sigma\sqrt{\tau}/2)N(d_{\alpha=2}) - N(d_{\alpha=1})^2\right]\right. \\ & -2XF_0\left[N(d_{\alpha=1}) - N(d_{\alpha=1})N(d_{\alpha=0})\right] \\ & \left.+X^2\left[N(d_{\alpha=0}) - N(d_{\alpha=0})^2\right]\right\}^{1/2} \end{aligned} \tag{2}$$

where

$$d_\alpha = F^\alpha \exp[(\alpha^2 - \alpha)\sigma\sqrt{\tau}/2] N\left(\frac{\ln F_0/X - \sigma^2\tau(1/2-\alpha)}{\sigma\sqrt{\tau}}\right) \tag{3}$$

the choice of the (slightly ugly) symbols $d_{\alpha=2}$ etc. is to suggest similarities to, but prevent confusion with d_1, d_2 from Black-Scholes