

# Useful formulas for Black76

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Using standard notations, namely for the expression of Greeks writing the partial derivative of  $F$  with respect to  $x$  as the subscript  $\frac{\partial F}{\partial x} = F_x$  but using standard variables names for the Black76 functions (which also contain subscripts: hopefully without confusion)

$$C(F, X, \tau) = \exp(-r\tau)(FN(d_1) - XN(d_2)) \quad (1)$$

$$P(F, X, \tau) = \exp(-r\tau)(XN(-d_2) - FN(-d_1)) \quad (2)$$

$$d_1 = \frac{\ln F/X + \sigma^2\tau/2}{\sigma\sqrt{\tau}} \quad d_2 = \frac{\ln F/X - \sigma^2\tau/2}{\sigma\sqrt{\tau}} \quad (3)$$

$$C_F = \exp(-r\tau)N(d_1)$$

$$C_X = -\exp(-r\tau)N(d_2)$$

$$C_{FF} = \exp(-r\tau) \frac{n(d_1)}{F\sigma\sqrt{\tau}}$$

$$C_{FF} = \frac{C_\sigma}{F^2\sigma\tau}$$

$$C_\sigma = \exp(-r\tau)F\sqrt{\tau}n(d_1)$$

$$C_\tau = -rC + \frac{1}{2}\sigma^2F^2C_{FF} \quad \text{Black76 PDE}$$

$$C_\tau = -rC + \frac{\exp(-r\tau)n(d_1)}{2\sqrt{\tau}}$$

$$C_{\sigma\sigma} = C_\sigma d_1 d_2 / \sigma \quad \text{“volga”}$$

$$C_{F\sigma} = -C_\sigma d_2 / (F\sigma\sqrt{\tau}) \quad \text{“vanna”}$$

$$C_{FF\sigma} = \frac{C_\sigma}{(F\sigma\sqrt{\tau})^2} (d_2^2 + \sigma\sqrt{\tau}d_2 - 1)$$

$$C_{FFF} = -\frac{C_\sigma}{F^3\sigma\tau} \left( \frac{d_2}{\sigma\sqrt{\tau}} + 1 \right)$$

$$C_{FFFF} = -\frac{C_\sigma}{F^4\sigma\tau(\sigma\sqrt{\tau})^2} (d_2^2 + 5d_2\sigma\sqrt{\tau} + 2(\sigma\sqrt{\tau})^2 - 1)$$

(4)

Inter-greek relations

$$\begin{aligned}C_{\sigma} &= F^2 \sigma \tau C_{FF} \\C &= FC_F + XC_X \\F^2 C_{FF} &= X^2 C_{XX} \\C_{FX} &= -\frac{X}{F} C_{XX} = -\frac{F}{X} C_{FF}\end{aligned}\tag{5}$$

Put-call relations

$$\begin{aligned}C(F, X) - P(F, X) &= \exp(-r\tau)(F - X) \quad \text{Put-call parity} \\C(F, X) &= P(X, F) \quad \text{Put-call symmetry}\end{aligned}\tag{6}$$

Probability densities

$$\begin{aligned}p(F(T)|F(0)) &= \frac{1}{\sqrt{2\pi}} \frac{1}{F(T)\sigma\sqrt{\tau}} \exp[-d_1^2/2] \\&= \exp(r\tau) C_{XX}(X = F(T), F = F(0)) \\&= \exp(r\tau) C_{FF}(F = F(T), X = F(0))\end{aligned}$$

Breeden & Litzenberger (7)